Consider a product $\alpha\beta$ where

$\alpha = a_1 g_1 + \cdots + a_m g_m$, $g_1 < \cdots < g_m$, $a_i \neq 0$ $(1 \leq i \leq m)$,

$\beta = b_1 h_1 + \cdots + b_n h_n$, $h_1 < \cdots < h_n$, $b_j \neq 0$ $(1 \leq j \leq n)$.

Choose i_0, j_0 such that $g_{i0} h_{j0}$ is *least* among $\{g_i h_j\}$.

Then $i_0 = 1$ (for otherwise $g_1 < g_{i0} \Rightarrow g_1 h_{j0} < g_{i0} h_{j0}$).

In particular, $g_{i0} h_{j0} = g_i h_j$ implies $i = i_0 = 1$ and hence $j = j_0$.

This shows that, in the product $\alpha\beta$, $a_1 b_{j0} g_1 h_{j0}$ cannot be "canceled out" by any other term, so $\alpha\beta \neq 0$.

Thus, it have to be a domain, and for the *J*-semisimplicity of *A* (when *G* in non *{1}*) we are done since any domain is J-semisimple.