

Consider a product $\alpha\beta$ where

$$\alpha = a_1g_1 + \cdots + a_mg_m, \quad g_1 < \cdots < g_m, \quad a_i \neq 0 \quad (1 \leq i \leq m),$$

$$\beta = b_1h_1 + \cdots + b_nh_n, \quad h_1 < \cdots < h_n, \quad b_j \neq 0 \quad (1 \leq j \leq n).$$

Choose i_0, j_0 such that $g_{i_0}h_{j_0}$ is *least* among $\{g_ih_j\}$. Then $i_0 = 1$ (for otherwise $g_1 < g_{i_0} \Rightarrow g_1h_{j_0} < g_{i_0}h_{j_0}$). In particular, $g_{i_0}h_{j_0} = g_ih_j$ implies $i = i_0 = 1$ and hence $j = j_0$. This shows that, in the product $\alpha\beta$, $a_1b_{j_0}g_1h_{j_0}$ cannot be “canceled out” by any other term, so $\alpha\beta \neq 0$. To compute $U(A)$, suppose $\alpha\beta = 1$. By the above consideration, we see that there is also a *largest* product among the g_ih_j ’s, which cannot be “canceled out” by other terms in the expansion of $\alpha\beta$. Thus, the only way for $\alpha\beta = 1$ to be possible is when $m = n = 1$, so A has only trivial units.