

Let $n = [G : Z(G)]$. Then G is n -abelian, i.e. $(xy)^n = x^n y^n$ for all $x, y \in G$. In particular, for any commutator $[a, b]$, we have $[a, b]^n = [a^n, b^n] = 1$ (since $a^n \in Z(G)$). Writing $G = \bigcup_{i=1}^n Z(G)a_i$ we see that $[G, G]$ is generated by the finite set $\{[a_i, a_j]\}$. So, $|[G, G]| < \infty$.