After enlarging the generating set, we may assume that $X := \{x_1, \ldots, x_n\}$ is closed under conjugation. Every element of G is a product of the x_i's. (We don't need their inverses since each x_i has finite order.) Fix a large number e such that $x_e^i = 1$ for all i. We finish by showing that every $a \in G$ is a product of no more than $n(e-1)$ elements from X. (This yields an explicit bound $|G| \le n^{n(e-1)}$.) Suppose $a = a_1 \cdots a_m$ where $a_i \in X$ and $m > n(e-1)$. It suffices to show that we can "reexpress" a with fewer factors. Since $m > n(e-1)$, some $x \in X$ must appear at least e times in the above expression. Say a_i is the first factor equal to x. Let $a'_j = x^{-1}a_jx \in X$ for $j < i$. Then $a = xa'_1 \cdots a'_{i-1}a_{i+1} \cdots a_m$.
Repeating this process, we can move e factors of x to the left, to get $a = x^e a^*_1 \cdots a^*_{i-1} \cdots a^*_{m-e}$, where $a^*_j \in X$. Since $x^e = 1$, we have now $a = a^*_1\, a^*_2 \cdots a^*_{m-e}$, as desired.