

Conditions

There are multiple solutions to the equation $5x = 2x^2 + 1$. Use logarithms to find their exact values.

How many solutions are there to the equation

$$2x = x^6?$$

Explain your reasoning. You do not need to find the solutions.

Solution

Consider:

$$5x = 2x^2 + 1$$

$$5x - 2x^2 = 1$$

$$\ln(5x - 2x^2) = 0$$

$$\ln(x(5 - 2x)) = 0$$

$$\ln x = \ln \frac{1}{5 - 2x}$$

$$5x - 2x^2 = 1$$

$$D = 25 - 8 = 17$$

$$x_{1,2} = \frac{5 \pm \sqrt{17}}{4}$$

How many solutions are there to the equation:

$$2x = x^6$$

This equation has two real-valued solutions:

$$x_1 = 0$$

$$x_2 = \sqrt[5]{2}$$