Evaluate the line integral [(2x-y+4)dx + (5y+3x-6)dy] around a circle of radius 4 with center at (0,0).

$$\int_L (2x - y + 4)\,dx + (5y + 3x - 6)dy$$

a) The equation of a circle of radius 4 with center at (0,0) in parametric form is

$$x = 2cost$$

$$y = 2sint$$

$$0 \leq t \leq 2\pi$$

$$dx = -2sintdt$$

$$dy = 2costdt$$

Hence

$$\int_L (2x - y + 4)\,dx + (5y + 3x - 6)dy =$$

$$= \int_0^{2\pi} (-(4cost - 2sint + 4)2sint + (10sint + 6cost - 6)2cost)dt =$$

$$= \int_0^{2\pi} (12costsint + 4sin^2x + 12cos^2x - 8sint - 12cost)dt$$

Using the double-angle formulae

$$cos2x = 2cos^2x - 1$$

$$\int_0^{2\pi} (12costsint + 4cos2t + 8 - 8sint - 12cost)dt =$$

$$= (6sin^2t + 2sin2t + 8t + 8cost - 12sint)\Big|_0^{2\pi} =$$

$$= 6 \times 0 + 2 \times 0 + 16\pi + 8 \times 1 - 12 \times 0 - 6 \times 0 - 2 \times 0 - 0 - 8 \times 1 + 12 \times 0 = 16\pi$$