Stepwise Solution of $2^{1/5}$ (Manually)

Newton's Method

Let z = final answer

y = the nth root you want to find which here is = 5

x = base = 2

1. Pick a level of approximation you are willing to live with, that is, a number 'e' > 0.
2. Let z(0) = our first guess of the answer.
3. Let n = 0.
4. Compute $$z(n+1) = \left(1 - \frac{1}{y}\right) z(n) + \left(\frac{x}{y * (z(n))^{y-1}}\right)$$
5. If $|z(n+1)-z(n)| < e$, stop and declare that z = z(n+1).
6. Replace n with n+1 and go to Step 3.

This is called Newton's Method, after Sir Isaac Newton. It converges on the right answer very quickly, and more so if your guess z(0) is a good one.

Solution of $2^{1/5}$

Let our first guess of the answer, z(0) = 1.2. We want our answer to be correct up to 4 decimal places so:

e = 0.0001 = 10^4.

For n = 0

$$z(1) = \left(1 - \frac{1}{5}\right) 1.2 + \left(\frac{2}{5 * (1.2)^{5-1}}\right)$$

⇨ z(1) = 1.152901

Now checking |z(1) – z(0)| = |1.152901 – 1.2| = 0.047 > e. So answer has not come yet.

For n = 1

$$z(2) = \left(1 - \frac{1}{5}\right) 1.152901 + \left(\frac{2}{5 * (1.152901)^{5-1}}\right)$$

⇨ z(2) = 1.14873

Now checking |z(2) – z(1)| = |1.14873 – 1.152901| = 0.00417 > e. So answer has not come yet.

For n = 2

$$z(3) = \left(1 - \frac{1}{5}\right) 1.14873 + \left(\frac{2}{5 * (1.14873)^{5-1}}\right)$$

⇨ z(3) = 1.14869

Now checking $|z(3) - z(2)| = |1.14869 - 1.14873| = 0.0000316 < e$. So stop now.

Answer is = 1.14869