First note that, kA has no nonzero nil ideals. Since kA is commutative, this simply means that the only nilpotent element of kA is zero. For $\alpha = \sum_{a \in A} \alpha_a a \in kA$, let $\alpha^* = \sum \alpha_a a^{-1}$. This defines an involution on kA, with $x\alpha = \alpha^* x$ for any $\alpha \in kA$. Any element $\sigma \in kG$ can be expressed uniquely in the form $\alpha + \beta x$, with $\alpha, \beta \in kA$. Let I be any nil ideal in kG, and let $\sigma \in I$. Then $(\alpha + \beta x)(\alpha^* + x\beta^*) = \alpha\alpha^* + \beta\beta^* + (\beta\alpha + \alpha\beta)x = \alpha\alpha^* + \beta\beta^* \in kA$.
Since this element is nilpotent, we must have $\alpha\alpha^* = \beta\beta^*$.
Therefore,
$(\alpha + \beta x)^2 = \alpha^2 + \beta x\beta x + \alpha\beta x + \beta x\alpha = \alpha^2 + \beta\beta^* + (\alpha\beta + \beta\alpha^*)x = \alpha^2 + \alpha\alpha^* + (\alpha\beta + \alpha^*\beta)x = (\alpha + \alpha^*)(\alpha + \beta x)$.
Say $(\alpha + \beta x)^n = 0$.
Then we have $0 = (\alpha + \alpha^*)^{n-1}(\alpha + \beta x)$, so that $(\alpha + \alpha^*)^{n-1}\alpha = 0$. But then $0 = x[(\alpha + \alpha^*)^{n-1}\alpha]x = (\alpha + \alpha^*)^{n-1}\alpha^*$.
Therefore, by addition, $(\alpha + \alpha^*)^n = 0$, so $\alpha = \alpha^*$. Now consider any $b \in A$. Then $b(\alpha + \beta x) \in I$ implies $b\alpha = (b\alpha)^*$. Suppose $\alpha \neq 0$; say α involves some group element $b-1 \in A$. Then $1 \in \text{supp}(b\alpha)$, and $b\alpha = (b\alpha)^*$ implies that $|\text{supp}(\alpha)| = |\text{supp}(b\alpha)|$ is odd, since A has no element of order 2. But if $\text{supp}(\alpha)$ *misses* some element $c-1 \in A$, then $1 \notin \text{supp}(c\alpha)$, and $c\alpha = (c\alpha)^*$ would imply that $|\text{supp}(\alpha)| = |\text{supp}(c\alpha)|$ is even. Therefore, we must have $\text{supp}(\alpha) = A$. If A is infinite, this is impossible. In this case, we conclude that $\alpha = 0$, and since $\sigma x = \beta x^2 = \beta$ is nilpotent, $\beta = 0$ too, so $\sigma = 0$. This completes the proof.