Next, assume $|A| < \infty$. Then $|A|$ is odd, so every element of A is a square. We continue to work with the element $\sigma = \alpha + \beta x \in I$. Write $\alpha = \sum_{a\in A} \alpha_a a \in kA$. For any $a_1, a_2 \in A$, choose $d \in A$ such that $d^2 = a_1^{-1}a_2^{-1}$. Then $da_1 = (da_2)^{-1}$, and $d\alpha = (d\alpha)^*$ implies that $\alpha_{a1} = \alpha_{a2}$. Therefore, $\alpha = \varepsilon \sum_{a\in A} a$ for some $\varepsilon \in k$. Since

$$(\alpha + \beta x)x = \beta + \alpha x \in I,$$

we have similarly $\beta = \varepsilon' \sum_{a\in A} a$. Now let $\tau = \sum_{g\in G} g \in Z(kG)$. We have $\tau^2 = |G|\tau = 2|A|\tau = 0$, so $k\tau$ is an ideal with $(k\tau)^2 = 0$. This implies that $k\tau \subseteq \text{rad } kG$. Applying the above analysis to $\sigma \in I := \text{rad } kG$, we have now $\sigma + \varepsilon'\tau = (\varepsilon + \varepsilon') \sum_{a\in A} a \in I$. Since this is a nilpotent element in kA, we conclude that $\varepsilon = \varepsilon'$, so $\sigma = \varepsilon\tau$. This shows that $I = k\tau$, with $I^2 = 0$.