Consider the homomorphism $\varphi : kG \to kS^2$ induced by the group surjection $G \to G/<(123)> = S2$. It is easy to see that $\ker(\varphi) = kG \cdot \alpha^1$, where $\alpha^1 = (123) - 1$. Since $kS^2 \sim k[t]/(t^2 - 1) \sim k \times k$ is semisimple, we have $J \subseteq \ker(\varphi)$. We claim that $J = \ker(\varphi)$ (whereby $kG/J \sim k \times k$). For this, it suffices to check that (123) acts as the identity on any left simple kG-module V . Now $V_0 = \{v \in V : (123)v = v\} \neq 0$ since $\alpha_1^3 = (123)^3 - 1 = 0 \in kG$ implies that $\alpha 1$ acts as a nilpotent transformation on V . It is easy to check that V_0 is a kG-submodule of V , so indeed $V_0 = V$. We have thus shown that

$J = \ker(\varphi) = \{x + y(123) + z(132) + u(12) + v(13) + w(23) : x + y + z = u + v + w = 0\}$.

and $kG/J \sim k \times k$