Define a map $\varepsilon : kG \to kG$ by $\varepsilon\left(\sum a_g g\right) = \sum a_g g^{-1}$. Since $(gh)^{-1} = h^{-1}g^{-1}$, and k is commutative, we can show that $\varepsilon(\alpha\beta) = \varepsilon(\beta)\varepsilon(\alpha)$. Of course ε is one-one, onto, and an additive homomorphism. Since we also have $\varepsilon^2 = 1$, ε is an involution on kG. If $I_1 \subset I_2 \subset \cdots$ was an ascending chain of right ideals in kG, $\varepsilon(I_1) \subset \varepsilon(I_2) \subset \cdots$
would have given an ascending chain of left ideals in kG. This gives the desired conclusion in the noetherian case.