In given case we have that, rad KG = (rad kG) $\otimes_k K$. Therefore,
(rad KG) $\cdot M^K$ = (rad $kG \otimes_k K$) $\cdot$ ($M \otimes_k K$)= (rad $kG \cdot M$) $\otimes_k K$.
It follows that: M is semisimple $\Leftrightarrow$ rad $kG \cdot M = 0 \Leftrightarrow$ (rad KG) $\cdot M^K = 0 \Leftrightarrow M^K$ is semisimple.