Let $A = k_0G/\text{rad } k_0G$. In fact, A can be any finite-dimensional semisimple k_0-algebra. ByWedderburn's Theorem, $A \sim \prod_i M_{n_i}(D_i)$, where the D_i's are finite-dimensional k_0-division algebras. By the other theorem of Wedderburn, each D_i is just a finite field extension of k_0. Now

$$A \otimes_{k_0} K \cong \left(\prod_i M_{n_i}(D_i)\right) \otimes_{k_0} K \cong \prod_i \left(M_{n_i}(D_i) \otimes_{k_0} K\right) \cong \prod_i M_{n_i}\left(D_i \otimes_{k_0} K\right) A \otimes_{k0} K$$

We are done if we can show that each $D_i \otimes_{k0} K$ is a finite direct product of fields. To simplify the notation, write D for D_i. The crux of the matter is that D/k_0 is a (finite) *separable* extension. Say $D = k_0(\alpha)$, and let $f(x)$ be the minimal polynomial of α over k_0. Then $f(x)$ is a *separable* polynomial over k_0. Over $K[x]$, we have a factorization $f(x) = f_1(x) \cdots f_m(x)$ where the f_i's are nonassociate irreducible polynomials in $K[x]$. By the Chinese Remainder Theorem:

$$D \otimes_{k_0} K \cong \frac{k_0[x]}{(f(x))} \otimes_{k_0} K \cong \frac{K[x]}{(f_1(x),\ldots,f_m(x))} \cong \prod_j \frac{K[x]}{(f_j(x))}$$

This is a finite direct product of finite (separable) field extensions of K, as desired.