

Let $R = kH \subseteq S = kG$, and fix a coset decomposition $G = \bigcup_{i \in I} H\sigma_i$. Then we have $S = \bigoplus_i R\sigma_i$. We may assume that some $\sigma_{i_0} = 1$. Therefore, ${}_R R = R\sigma_{i_0}$ is a direct summand of ${}_R S$. Similarly, R_R is a direct summand of S_R . So, we get the two desired conclusions.