

Fixing a k -homomorphism $f: V \rightarrow W$ such that f / W is the identity, we can define $g: V \rightarrow V$ as

$$g(v) = |G|^{-1} \sum_{g \in G} \sigma_i^{-1} f(\sigma v) \text{ for } v \in V.$$

We check easily that g is a kG -homomorphism with $g / W = \text{Id}_W$, and so $V = W \oplus \ker(g)$.