

Fixing a  $k$ -homomorphism  $f: V \rightarrow W$  such that  $f|_W$  is the identity, we can define  $g: V \rightarrow V$  as

$$g(v) = |G|^{-1} \sum_{g \in G} \sigma_g^{-1} f(\sigma_g v) \text{ for } v \in V.$$

We check easily that  $g$  is a  $kG$ -homomorphism with  $g|_W = \text{Id}_W$ , and so  $V = W \oplus \ker(g)$ .