

Let $k = \mathbb{F}_p$ and let G act on A by conjugation. The subgroup $C_G(A)$ is also normal in G , and acts trivially on A . Writing $G' = G/C_G(A)$, we may therefore view A as a kG' -module. Since $|G'| = [G : C_G(A)]$ is prime to $p = \text{char } k$, kG' is a semisimple ring. The assumption that $B \triangleleft G$ implies that B is a kG' -submodule of A . Therefore, $A = B \oplus C$ for a suitable kG' -submodule $C \subseteq A$. Going back to the multiplicative notation, we have $A = B \times C$, and $C \triangleleft G$.