Fix a coset decomposition $G = \bigcup_{i=1}^{n} H\sigma_i$. Let W be any kGsubmodule of V. Let $f : V \to W$ be a kH-homomorphism with $f|W = 1_W$. Here we try to modify f into a kG-homomorphism $g : V \to W$ with $g|W = 1_W$. Define

$g(v) = n^{-1}\sum_{i=1}^{n} \sigma_i^{-1} f(\sigma_i v)$, where($v \in V$).

It is clear that $g|W = 1_W$, so we only need to prove $g(\tau v) = \tau g(v)$ for every $\tau \in G$. Write $\sigma_i\tau = \theta_i\sigma_{\alpha(i)}$ where $\theta_i \in H$ and α is a permutation of $\{1, \dots, n\}$. Then

$$g(\tau v) = n^{-1}\sum_{i=1}^{n} \sigma_i^{-1} f(\sigma_i \tau v) = n^{-1}\sum_{i=1}^{n} \sigma_i^{-1} f\left(\theta_i \sigma_{\alpha(i)} v\right) = n^{-1}\sum_{i=1}^{n} \sigma_i^{-1}\theta_i f\left(\sigma_{\alpha(i)} v\right) =$$

$$= n^{-1}\sum_{i=1}^{n} \tau\sigma_{\alpha(i)}^{-1} f\left(\sigma_{\alpha(i)} v\right) = \tau\left(n^{-1}\sum_{i=1}^{n} \sigma_{\alpha(i)}^{-1} f\left(\sigma_{\alpha(i)} v\right)\right) = \tau g(v)$$