

For  $A, B \in S$ , we have  $A \cap B \in S$ . Thus,  $\inf\{A, B\}$  is given simply by  $A \cap B$ . For  $\sup\{A, B\}$ , we take  $\sqrt{A+B} \in S$ , where the “radical” of an ideal. A semiprime ideal  $C$  contains both  $A$  and  $B$  iff  $C \supseteq A+B$ , iff  $C \supseteq \sqrt{A+B}$ . Thus,  $\sqrt{A+B}$  is indeed the supremum of  $A$  and  $B$  in  $S$ . This shows that  $S$  is a lattice. Clearly,  $S$  has a largest element,  $R$ , and a smallest element,  $\text{Nil} \neq R$ .