Question 1. *Show that*

$$R = \begin{pmatrix} Z & nZ \\ 0 & Z \end{pmatrix}$$

is not a prime ring.

Solution. Recall that a ring is called prime if it is non-zero and for any $a, b \in R$ the equality $aRb = 0$, implies $a = 0$ or $b = 0$. Take

$$a = b = \begin{pmatrix} 0 & n \\ 0 & 0 \end{pmatrix} \in R.$$

Obviously, a and b are nonzero. But for any

$$r = \begin{pmatrix} x & nz \\ 0 & y \end{pmatrix} \in R$$

we have

$$arb = \begin{pmatrix} 0 & n \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x & nz \\ 0 & y \end{pmatrix} \begin{pmatrix} 0 & n \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & ny \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & n \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

which is zero of R. Thus, $arb = 0$ for all $r \in R$, while $a, b \neq 0$. By definition, R is not prime. □