Conditions

lim x->0 f(x) if |f(x)-5|<3x^2

Solution

We must consider

$$\lim_{x \to 0} f(x)$$

where

$$|f(x) - 5| < 3x^2$$

Let's remind the definition of a limit F of a function $f(x)$ at a point x_0:

$$\forall \varepsilon > 0 \ \exists \delta = \delta(\varepsilon) > 0 : \ \forall x : |x - x_0| < \delta \ |f(x) - F| < \varepsilon$$

I claim, that the limit of our function at point 0 is equal to 5. Let me prove this:

Fix $\varepsilon > 0$, now I have to find $\delta(\varepsilon)$: $\forall x$: $|x| < \delta \ |f(x) - 5| < \varepsilon$

Consider:

$$|f(x) - 5|$$

According to conditions:

$$|f(x) - 5| < 3x^2 < 3\delta^2$$

Here it's enough to take $\delta = \sqrt{\frac{\varepsilon}{3}}$

So, we have:

$$\forall \varepsilon > 0 \ \exists \delta = \sqrt{\frac{\varepsilon}{3}} > 0 : \ \forall x : |x| < \sqrt{\frac{\varepsilon}{3}} \ |f(x) - 5| < \varepsilon$$

The claim is proved.

Answer: The limit is equal to 5