Question 1. *Prove that if A is square and there exist matrices B and C such that $AB = I$ and $CA = I$, then $B = C$ and A is invertible.*

Solution. Since A and I are square and $AB = CA = I$, then B and C should be square of the same size. Multiplying $AB = I$ by C on the left, using the associativity of product and the fact that I is the identity matrix, we get

$$CAB = C.$$

Similarly $CA = I$, being multiplied by B on the right, gives

$$CAB = B.$$

Thus, $B = C$. Therefore, $AB = BA = I$ and hence B is the inverse of A. $\square$