Let $A = [a]_{mn}, B = [b]_{np}$. Let $AB = [c]_{mp}$. Then from the definition of matrix product: $\forall i \in [1,...,m], j \in [1,..., p]: c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$. So, let $(AB)^t = [r]_{pm}$. The dimensions are correct, because AB is an $m \times p$ matrix, thus making $(AB)^t$ a $p \times m$ matrix. Thus: $\forall i \in [1,...,m], j \in [1,..., p]: r_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$. Now, let $B^t A^t = [s]_{mp}$. Thus: $\forall i \in [1,...,m], j \in [1,..., p]: r_{ji} = \sum_{k=1}^{n} b_{kj} a_{ik}$.

As the underlying structure of A and B is a commutative ring, then $a_{ik} \circ b_{kj} = b_{kj} \circ a_{ik}$. Note the order of the indices in the term in the summation sign on the RHS of the above. They are reverse what they would normally be because we are multiplying the transposes together. Thus it can be seen that $r_{ji} = s_{ji}$ and the result follows.