Question 1. *Show that for each* $k = 1, \ldots, K$*,* $Col_k(C)$*, the* k*-th column of the matrix* $C = AB$*, is* $Col_k(C) = ACol_k(B)$*.*

Solution. Recall that $Col_k(C)$ consists of the elements $C(i, k)$ of the matrix C, $i = 1, \ldots, K$. By definition of the product of matrices:

$$C(i, k) = (AB)(i, k) = \sum_{j=1}^{K} A(i, j)B(j, k).$$

Thus, $C(i, k)$ is obtained by "multiplication" of the i-th row of A by the k-th column $Col_k(B)$ of B. Therefore, $Col_k(C)$ is the result of applying A to the vector $Col_k(B)$. $\square$