

We first make a general observation on the semiprime ideals in a right artinian ring  $R$ . Say  $A \subseteq R$  is semiprime. Then  $\text{rad } R \subseteq A$  since  $\text{rad } R$  is nilpotent. On the other hand, if  $A$  is any ideal containing  $\text{rad } R$ , then  $A/\text{rad } R$  must be the sum of some simple components of the semisimple ring  $R/\text{rad } R$ . But then  $R/A$  is semisimple and hence  $A$  is automatically semiprime. On the other hand, for such  $A$  to be prime, we need  $A/\text{rad } R$  to be the sum of all but one simple components of  $R/\text{rad } R$ . To apply this to the ring  $R$  in the exercise, recall that  $\text{rad } R$  is the ideal of  $R$  consisting of matrices with zero diagonal, and  $R/\text{rad } R \cong k \times k \times k$ . In particular, the prime (or maximal) ideals of  $R$  are:

$$\begin{pmatrix} 0 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{pmatrix}, \begin{pmatrix} k & k & k \\ 0 & 0 & k \\ 0 & 0 & k \end{pmatrix}, \begin{pmatrix} k & k & k \\ 0 & k & k \\ 0 & 0 & 0 \end{pmatrix}$$

There are only five more semiprime ideals in  $R$ , namely:

$$\begin{pmatrix} 0 & k & k \\ 0 & 0 & k \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k & k & k \\ 0 & 0 & k \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & k & k \\ 0 & k & k \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & k & k \\ 0 & 0 & k \\ 0 & 0 & k \end{pmatrix}, \text{ and } R.$$