Question 1. *Show that a nonzero central element of a prime ring R is not a zero-divisor in R.*

Solution. Recall that a ring is called prime if it is non-zero and for any $a, b \in R$ the equality $aRb = 0$, implies $a = 0$ or $b = 0$. Suppose a is a central element of R and $ab = 0$ for some $b \in R$. Then for any $c \in R$ we have

$$acb = (ac)b = (ca)b = c(ab) = c \cdot 0 = 0.$$

So, $aRb = 0$ and hence $a = 0$ or $b = 0$, since R is prime. Thus, a is not a divisor of zero. □