

**Question 1.** Prove that  $0v = 0$  for all  $v \in V$ , and use this to prove that  $(-1)v = -v$  for all  $v \in V$ .

*Solution.* By distributivity of scalar multiplication with respect to field addition

$$0v = (0 + 0)v = 0v + 0v.$$

Subtracting  $0v$  from both sides of this equality we get  $0v = 0$ .

Furthermore, it follows from the axiom  $1v = v$  that

$$v + (-1)v = 1v + (-1)v = (1 - 1)v = 0v = 0.$$

This means that  $(-1)v = -v$ . □