

Question 1. *Prove that $0v = 0$ for all $v \in V$, and use this to prove that $(-1)v = -v$ for all $v \in V$.*

Solution. By distributivity of scalar multiplication with respect to field addition

$$0v = (0 + 0)v = 0v + 0v.$$

Subtracting $0v$ from both sides of this equality we get $0v = 0$.

Furthermore, it follows from the axiom $1v = v$ that

$$v + (-1)v = 1v + (-1)v = (1 - 1)v = 0v = 0.$$

This means that $(-1)v = -v$.

□