Evaluate the integral between (1,1) and (4,2): [(x+y)dx + (y-x)dy] Along
a) a straight line.
b) straight lines from (1,1) to (1,2) and then to (4,2).
c) the curve x = 2t^2 + t + 1, y = t^2 +1.

$$\int_L (x+y)\,dx + (y-x)dy$$

a) The equation of the straight line passing through the points (1,1) and (4,2) is $\frac{x-1}{4-1} = \frac{y-1}{2-1}$, so

$y = \frac{1}{3}x + \frac{2}{3}$ and $dy = \frac{1}{3}dx$, hence

$$\int_L (x+y)\,dx + (y-x)dy = \int_1^4 \left(\left(x + \frac{1}{3}x + \frac{2}{3}\right) + \left(\frac{1}{3}x + \frac{2}{3} - x\right) \times \frac{1}{3}\right) dx =$$

$$= \int_1^4 \left(\frac{10}{9}x + \frac{8}{9}\right) dx = \left(\frac{5}{9}x^2 + \frac{8}{9}x\right)\Bigg|_1^4 = \left(\frac{80}{9} + \frac{32}{9} - \frac{5}{9} - \frac{8}{9}\right) = 11$$

b) The equation of the straight line passing through the points (1,1) and (1,2) is $x = 1$, so $dx = 0$

The equation of the straight line passing through the points (1,2) and (4,2) is $y = 2$, so $dy = 0$

Hence

$$\int_L (x+y)\,dx + (y-x)dy = \int_1^2 (y-1)dy + \int_1^4 (x+2)dx = \left(\frac{y^2}{2} - y\right)\Bigg|_1^2 + \left(\frac{x^2}{2} + 2x\right)\Bigg|_1^4 =$$

$$= \left(\frac{4}{2} - 2 - \frac{1}{2} + 1\right) + \left(\frac{16}{2} + 8 - \frac{1}{2} - 2\right) = 14$$

c) $x = 2t^2 + t + 1 \;\; y = t^2 + 1$

$dx = (4t+1)dt$ and $dy = 2tdt$

When (1,1), then $2t^2 + t + 1 = 1$ and $t^2 + 1 = 1$, so $t = 0$

When (4,2), then $2t^2 + t + 1 = 4$ and $t^2 + 1 = 2$, so $t = 1$

$$\int_L (x+y)\,dx + (y-x)dy = \int_0^1 \left((2t^2 + t + 1 + t^2 + 1)(4t+1) + (t^2 + 1 - 2t^2 - t - 1)2t\right)dt =$$

$$= \int_0^1 (10t^3 + 5t^2 + 9t + 2)dt = \left(\frac{5t^4}{2} + \frac{5t^3}{3} + \frac{9t^2}{2} + 2t\right)\Bigg|_0^1 = \left(\frac{5}{2} + \frac{5}{3} + \frac{9}{2} + 2\right) = 10\frac{2}{3}$$