Question 1. *The angle of elevation of the top of a tree is found to be* 33° *at one point and* 59° *at a point* 31 *ft. nearer the tree. How high is the tree if both observation points and the base of the tree are in the same horizontal plane?*

Solution. Let A be the top of the tree, B its bottom, C_1 and C_2 the points from which the top is seen under the angles 33° and 59°, respectively. It is given that $\angle AC_1B = 33^\circ$, $\angle AC_2B = 59^\circ$ and $C_1C_2 = 31$. Therefore, $\angle AC_2C_1 = 180^\circ - 59^\circ = 121^\circ$ and $\angle C_1AC_2 = 180^\circ - 121^\circ - 33^\circ = 26^\circ$. Use the law of sines for $\triangle AC_1C_2$:

$$\frac{C_1C_2}{\sin \angle C_1AC_2} = \frac{AC_2}{\sin \angle AC_1C_2},$$

and hence

$$AC_2 = C_1C_2 \frac{\sin \angle AC_1C_2}{\sin \angle C_1AC_2} = 31 \frac{\sin 33^\circ}{\sin 26^\circ}.$$

Then from $\triangle ABC_2$ we have

$$AB = AC_2 \sin \angle AC_2B = 31 \frac{\sin 33^\circ}{\sin 26^\circ} \cdot \sin 59^\circ \approx 33.$$

Answer: approximately 33 ft. □