

Write the equation of the line through the given point and with the given slope. Give your answer in the form $y = mx + b$: $(4,9)$, $m = \frac{1}{4}$

Solution:

We start with the general equation of a straight line $y = mx + b$. A common form of a linear equation in the two variables x and y , where m and b designate constants. The origin of the name "linear" comes from the fact that the set of solutions of such an equation forms a straight line in the plane. In this particular equation, the constant m determines the slope or gradient of that line, and the constant term b determines the point at which the line crosses the y - axis, otherwise known as the y - intercept.

Suppose that we want to find the equation of a line which has a gradient of $\frac{1}{4}$ and passes through the point $(4,9)$. Whilst we know the gradient, we do not know the value of the y -intercept b . We know the gradient is $\frac{1}{4}$ and so we can substitute this value for m straightaway. This gives $y = \frac{1}{4}x + b$. We now use the fact that the line passes through the point $(4,9)$. This means that when $x = 4$, y must be 9. Substituting these values we find:

$$9 = \frac{1}{4} \cdot 4 + b \Rightarrow b + 1 = 9, b = 8. \text{ So, we find the value of the } y\text{-intercept } b = 8.$$

$$\text{So the equation of the line is } y = \frac{1}{4}x + 8$$

Also we can solve applying same method.

We can work out a general formula for problems of this type by using the same method. We shall take a general line with gradient $m = \frac{1}{4}$, passing through the fixed point $(4,9)$, where $x_1 = 4, y_1 = 9$. Use the general equation of a straight line $y = mx + b$. now use the fact that the line passes through $(4,9)$. This means that when $x = x_1$, y must be y_1 . Substituting these values we find $y_1 = mx_1 + b, \Rightarrow b = y_1 - mx_1$. So the equation of the line is $y = mx + y_1 - mx_1$. We can write this in the alternative form $y - y_1 = m(x - x_1)$.

This then represents a straight line with gradient m , passing through the point . So this general form is useful if we know the gradient and one point on the line. In our case, substitute in the values straight away:

$$y - 9 = \frac{1}{4}(x - 4)$$

$$y - 9 = \frac{1}{4}x - 1$$

$$y = \frac{1}{4}x + 8$$

Answer:

The equation of the line through the given point $(4,9)$, and with the given slope $m = \frac{1}{4}$, is $y = \frac{1}{4}x + 8$

Graph of the equation

