Task:

$v\frac{dv}{dx} = -g\frac{R^2}{x^2}$, solve for t for any given x and $g = 9.81\frac{m}{s^2}$

Solution:

Then:

$$v\frac{dv}{dx} = -g\frac{R^2}{x^2}$$

$$v = \frac{dx}{dt}$$

$$\frac{dx}{dt}\frac{dv}{dx} = -g\frac{R^2}{x^2}$$

$$\frac{dv}{dt} = -g\frac{R^2}{x^2}$$

$$a = -g\frac{R^2}{x^2}$$

$$x^2 = -\frac{gR^2}{a}$$

$$x = \pm\sqrt{-\frac{gR^2}{a}}$$

$$f(t) = \pm\sqrt{-\frac{gR^2}{a}}$$

Depending on f(t) answers will differ.

For example, let $x = f(t) = \frac{at^2}{2} + v(0)t + x(0)$

$$\frac{at^2}{2} + v(0)t + x(0) = \pm\sqrt{-\frac{gR^2}{a}}$$

$$\frac{at^2}{2} + v(0)t + x(0) \mp \sqrt{-\frac{gR^2}{a}} = 0$$

$$D = v^2(0) - 2at^2\left(x(0) \mp \sqrt{-\frac{gR^2}{a}}\right)$$

$$t_1 = \frac{-v(0) + \sqrt{v^2(0) - 2at^2\left(x(0) \mp \sqrt{-\frac{gR^2}{a}}\right)}}{a}$$

$$t_2 = \frac{-v(0) - \sqrt{v^2(0) - 2at^2\left(x(0) \mp \sqrt{-\frac{gR^2}{a}}\right)}}{a}$$

$t \geq 0$

Answer:

Depending on f(t) answers will differ

In case of $x = f(t) = \frac{at^2}{2} + v(0)t + x(0)$

$$t_1 = \frac{-v(0) + \sqrt{v^2(0) - 2at^2\left(x(0) \mp \sqrt{-\frac{gR^2}{a}}\right)}}{a}$$

$$t_2 = \frac{-v(0) - \sqrt{v^2(0) - 2at^2\left(x(0) \mp \sqrt{-\frac{gR^2}{a}}\right)}}{a}$$

$t \geq 0$