Question 1. *How is the value of the determinant related to whether a matrix is singular or non-singular?*

Solution. By definition a square matrix is called singular if it is not invertible, i. e. if it does not have the inverse matrix. There is a criterion of being singular in terms of determinant: a matrix is singular if and only if its determinant is zero. For example, the determinant of

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

equals 1, so A should be nonsingular. Indeed, one can easily see that

$$A^{-1} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}.$$

But, for instance

$$B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

is singular, because its determinant is zero.
Answer: a matrix in singular exactly when its determinant is zero. □