

i) Given that  $\cos(x+30)=3\cos(x-30)$ , prove that  $\tan x = -\frac{\sqrt{3}}{2}$

$$\cos(x+30)=3\cos(x-30) \Leftrightarrow \cos x \cdot \cos 30 - \sin x \cdot \sin 30 = 3(\cos x \cdot \cos 30 + \sin x \cdot \sin 30) \Leftrightarrow$$

Solution: 
$$\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \frac{3\sqrt{3}}{2} \cos x + \frac{3}{2} \sin x \Leftrightarrow \sqrt{3} \cos x = -2 \sin x \Leftrightarrow \tan x = \frac{-\sqrt{3}}{2}$$

(ii) prove that  $\frac{1-\cos 2x}{\sin 2x} = \tan x$

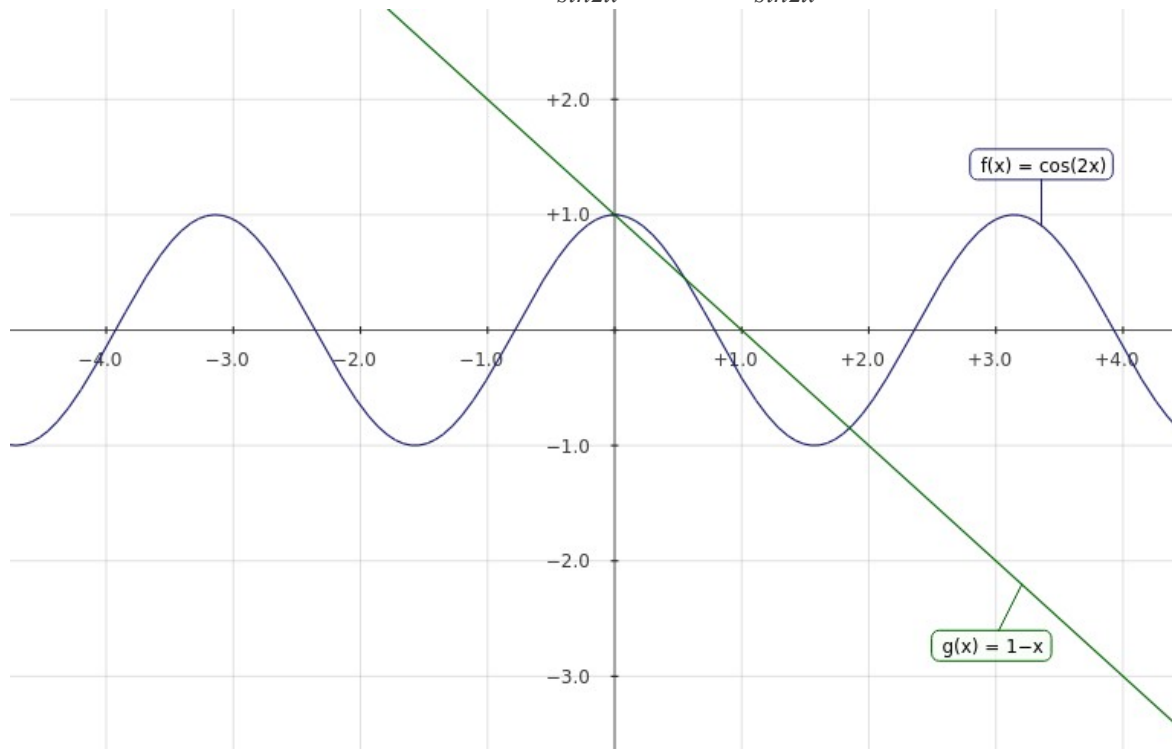
Solution: 
$$\frac{1-\cos 2x}{\sin 2x} = \frac{1-1-2\sin^2 x}{2\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$$

(iii) verify that  $x = 180$  is a solution of the equation  $2x = 2 - 2\cos 2x$

Solution:  $x = 180^\circ = \pi, 2x = 2 - 2\cos 2x \Leftrightarrow x = 1 - \cos 2x \Leftrightarrow \pi = 1 - \cos(2\pi) \Leftrightarrow \pi = 1 - 1 = 0 \Leftrightarrow \pi \neq 0$

(iv) Using the result from part (ii) or otherwise find the two other solutions  $0 < x < 360$  of the equation  $2x = 2 - 2\cos 2x$

Solution:  $2x = 2 - 2\cos 2x \Leftrightarrow x = 1 - \cos 2x, \frac{1-\cos 2x}{\sin 2x} = \tan x \Leftrightarrow \frac{x}{\sin 2x} = \tan x$



$g(x) = 1 - x, f(x) = \cos(2x), 0^\circ < x < 360^\circ$ , that's why there are 2 solutions.