

prove that  $\frac{\sin(5x) - 2\sin(3x) + \sin(x)}{\cos(5x) - \cos(x)} = \tan x$

$$\frac{\sin(5x) - 2\sin(3x) + \sin(x)}{\cos(5x) - \cos(x)} = \tan(x)$$

$$\begin{aligned} \frac{\sin(5x) - 2\sin(3x) + \sin(x)}{\cos(5x) - \cos(x)} &= \frac{2\sin(3x)\cos(2x) - 2\sin(3x)}{-2\sin(3x)\sin(2x)} = \frac{2\sin(3x)(\cos(2x) - 1)}{-2\sin(3x)\sin(2x)} = \\ &= \frac{1 - \cos(2x)}{\sin(2x)} = \frac{\cos^2 x + \sin^2 x - \cos^2 x + \sin^2 x}{2\sin x \cos x} = \frac{2\sin^2 x}{2\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x \end{aligned}$$

$$\tan x = \tan x$$

Proved!