

$$\int \frac{y-5}{y^2+5y+6} dy = \dots$$

$$\frac{y-5}{y^2+5y+6} = \frac{y-5}{(y+2)(y+3)} = \frac{A}{y+2} + \frac{B}{y+3} = \frac{(A+B)y + (3A+2B)}{(y+2)(y+3)}$$

$$\begin{cases} A+B=1 \\ 3A+2B=-5 \end{cases}$$

$$\begin{cases} 2A+2B=2 \\ 3A+2B=-5 \end{cases}$$

$$-A=7$$

$$A=-7, B=8$$

$$\frac{y-5}{y^2+5y+6} = \frac{-7}{y+2} + \frac{8}{y+3}$$

$$\dots = \int \left(\frac{-7}{y+2} + \frac{8}{y+3} \right) dy = -7 \ln|y+2| + 8 \ln|y+3| + C, C = \text{const}$$