

Find the integral: $\int \frac{2x+7}{x^3-3x^2+2x} dx$.

Solution:

$$\frac{2x+7}{x^3-3x^2+2x} = \frac{2x+7}{x(x^2-3x+2)} = \frac{2x+7}{x(x-2)(x-1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-1}$$

$$A = \frac{2x+7}{(x^2-3x+2)} \Big|_{x=0} = \frac{7}{2}$$

$$B = \frac{2x+7}{x(x-1)} \Big|_{x=2} = \frac{11}{2}$$

$$C = \frac{2x+7}{x(x-2)} \Big|_{x=1} = -9$$

$$\begin{aligned} \int \frac{2x+7}{x^3-3x^2+2x} dx \\ = \int \left(\frac{7}{2} \frac{1}{x} + \frac{11}{2} \frac{1}{x-2} - \frac{9}{x-1} \right) dx = \frac{7}{2} \ln|x| + \frac{11}{2} \ln|x-2| \\ - 9 \ln|x-1| + C, \end{aligned}$$

where $C = \text{const.}$

Answer: $\int \frac{2x+7}{x^3-3x^2+2x} dx = \frac{7}{2} \ln|x| + \frac{11}{2} \ln|x-2| - 9 \ln|x-1| + C.$