if tanx/2=secz, prove that $\sin x = \frac{1-\tan\frac{z}{2}}{1+\tan\frac{z}{2}}$

Solution:

$$\sin x = \frac{\sin x}{1} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}} = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

if $\tan\frac{x}{2} = \sec z$, then

$$\sin x = \frac{2\sec z}{1+\sec z^2} = 2\frac{\frac{1}{\cos z}}{1+\left(\frac{1}{\cos z}\right)^2} = \frac{2\cos z}{1+\cos z^2}$$

We use that

$$\cos z = \frac{1-\tan\frac{z^2}{2}}{1+\tan\frac{z^2}{2}}$$

Then

$$\sin x = \frac{2\frac{1-\tan\frac{z^2}{2}}{1+\tan\frac{z^2}{2}}}{1+\left(\frac{1-\tan\frac{z^2}{2}}{1+\tan\frac{z^2}{2}}\right)^2} = \frac{2\left(1-\tan\frac{z^2}{2}\right)\left(1+\tan\frac{z^2}{2}\right)}{\left(1+\tan\frac{z^2}{2}\right)^2+\left(1-\tan\frac{z^2}{2}\right)^2} = \frac{2(1-\tan\frac{z^4}{2})}{2(1+\tan\frac{z^4}{2})} = \frac{1-\tan\frac{z^4}{2}}{1+\tan\frac{z^4}{2}}$$

Now we have a trouble, because $\tan\frac{z}{2} \neq \tan\frac{z^4}{2}$. And right identity is $\sin x = \frac{1-\tan\frac{z^4}{2}}{1+\tan\frac{z^4}{2}}$.