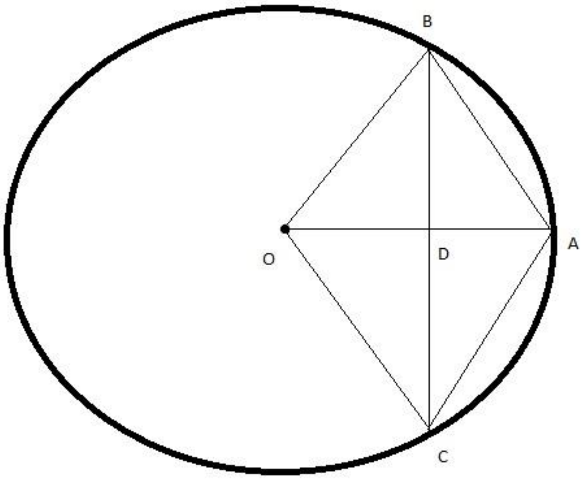
Conditions

In a circle with a 12 inch radius, find the legnth of a segment joining the midpoint of a 20 inch chord and the center of the circle

Solution

Let's consider a graph:



As we can see, BC is a chord and its length is 20 inches. OA is a radius, which is intersect BC in a point D with an angle 90 degrees (we can build radius in this way). Its length is 12 inches. It's obvious to notice, that BD=DC, OB=OC, AC=AB. But OD doesn't equal to AD.

As BC is a chord, than we know a formula which links the radius, the chord length and the angle BOC:

$$BC = 2 \cdot OA \cdot \sin\frac{\angle BOC}{2}$$

So we can find this angle now:

$$20 = 2 \cdot 12 \cdot \sin\frac{\angle BOC}{2}$$

$$\sin\frac{\angle BOC}{2} = \frac{20}{24} = \frac{5}{6}$$

$$\angle BOC = 2\sin^{-1}\frac{5}{6}$$

Then, we can find the answer on the question in the task. Consider, for example, triangle ODB. Angle O is a half of angle BOC and it's $\sin^{-1}\frac{5}{6}$. As this is a right triangle, we know that:

$$OD = \frac{BD}{\tan\left(\sin^{-1}\frac{5}{6}\right)} = \frac{\mathbf{10}}{\mathbf{tan}\left(\mathbf{sin}^{-1}\frac{\mathbf{5}}{\mathbf{6}}\right)}$$