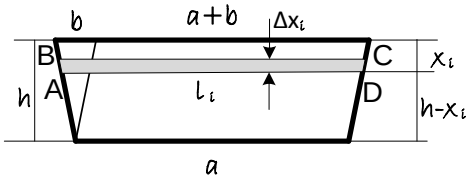
If the water face of a dam is in the shape of a trapezoid.

Height =(h), width at water level =(a+b) and width at base = (a)

How can you show the resultant force = 1/6wh^2 (3a+b) ?



The force on i-th strip is $F_i = w * x_i * S_i$

Where S_i is the area of ABCD, for practical purposes we suppose it to be a rectangle, so

$S_i = l_i * \Delta x_i$

$$\frac{l_i - a}{h - x_i} = \frac{b}{h}$$

$l_i = a + b - \frac{b*x_i}{h}$, so $S_i = (a + b - \frac{b*x_i}{h}) * \Delta x_i$ and $F_i = w * x_i * (a + b - \frac{b*x_i}{h}) * \Delta x_i$

The resultant force is

$$F = \int_0^h wx(a + b - \frac{bx}{h})dx = w\left(\left(\frac{(a+b)x^2}{2} - \frac{bx^3}{3h}\right)\right)\Bigg|_0^h = w\left(\frac{(a+b)h^2}{2} - \frac{bh^2}{3}\right) = \frac{wh^2}{6}(3a - b)$$