Question 1. *Let* $\phi_i : G_i \to G_1 \times G_2 \times G_3 \times \cdots \times G_i \times \cdots \times G_r$ *be given by* $\phi_i(g_i) = (e_1, e_2, \ldots, g_i, \ldots, e_r)$ *where* $g_i \in G_i$ *and* e_j *is the identity of* G_j. *Prove that this is an injective map.*

Solution. Suppose $\phi_i(g_i) = \phi_i(h_i)$ for some $g_i, h_i \in G_i$. By definition of ϕ_i this means that

$$(e_1, e_2, \ldots, g_i, \ldots, e_r) = (e_1, e_2, \ldots, h_i, \ldots, e_r),$$

i. e.

$$\begin{aligned} e_1 &= e_1, \\ e_2 &= e_2, \\ &\cdots \\ g_i &= h_i, \\ e_r &= e_r. \end{aligned}$$

In particular, $g_i = h_i$. Thus, $\phi_i(g_i) = \phi_i(h_i)$ implies $g_i = h_i$, and so ϕ_i is injective, $i = 1, \ldots, r$. $\square$