First note that, for $i \geq 1$, $1 + J^i$ is a normal subgroup of $U(R)$, as it is the kernel of the natural group homomorphism $U(R) \to U(R/J^i)$. We define a map $\sigma : J^i \to (1 + J^i)/(1 + J^{i+1})$ by $\sigma(x) = (1+x) \cdot (1 + J^{i+1})$, for every $x \in J^i$. This is a group homomorphism since, for $x, y \in J^i$: $\sigma(x)\sigma(y) = (1 + x)(1 + y) \cdot (1 + J^{i+1}) = (1+x + y + xy) \cdot (1 + J^{i+1}) = (1+x + y)[1 + (1 + x + y)^{-1}xy] \cdot (1 + J^{i+1}) = \sigma(x + y)$.
Since σ is surjective and has kernel J^{i+1}, it induces a group isomorphism $J^i/J^{i+1} \sim (1 + J^i)/(1 + J^{i+1})$.