We construct here a *cyclic* artinian left module M of infinite length over some (noncommutative, non left-noetherian) ring R. In the triangular ring $R = \begin{pmatrix} \mathbb{Q} & 0 \\ \mathbb{Q} & \mathbb{Z} \end{pmatrix}$, the idempotent $e = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ generates the left ideal $Re = \begin{pmatrix} \mathbb{Q} & 0 \\ \mathbb{Q} & 0 \end{pmatrix}$ (which is in fact an ideal). We express this module in the simpler form $\begin{pmatrix} \mathbb{Q} \\ \mathbb{Q} \end{pmatrix}$, and consider its submodule $\begin{pmatrix} 0 \\ \mathbb{Z}_{(p)} \end{pmatrix}$, where $Z_{(p)}$ denotes the localization of Z at a prime ideal (p).

Since $\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ cx & 0 \end{pmatrix}$ (a, $b \in$ Q; $c \in$ Z), the ideal Re acts trivially on $\begin{pmatrix} 0 \\ \mathbb{Q} \end{pmatrix}$ so the R-submodules of $\begin{pmatrix} 0 \\ \mathbb{Q} \end{pmatrix}$ are just $\begin{pmatrix} 0 \\ G \end{pmatrix}$ where G is any subgroup of Q. Now $Q/Z_{(p)}$ is isomorphic to the Prufer p-group (the group of p^n-th roots of unity for $n \in$ N), which is of infinite length as a Z-module. Therefore, the cyclic R-module $M := \begin{pmatrix} \mathbb{Q} \\ \mathbb{Q} \end{pmatrix} \Big/ \begin{pmatrix} 0 \\ \mathbb{Z}_{(p)} \end{pmatrix} \supseteq M' = \begin{pmatrix} 0 \\ \mathbb{Q} \end{pmatrix} \Big/ \begin{pmatrix} 0 \\ \mathbb{Z}_{(p)} \end{pmatrix}$ is also of infinite length. Now M/M' ~ $\begin{pmatrix} \mathbb{Q} \\ \mathbb{Q} \end{pmatrix} \Big/ \begin{pmatrix} 0 \\ \mathbb{Q} \end{pmatrix}$ is a *simple* R-module, and M' is an *artinian* Z-module (and hence an artinian R-module). It follows that M is also an artinian R-module (of infinite length), as desired.