

solve the following equation in the given interval :

$$\sec 2x = 3\cos 2x + \cos(90-2x) ; -90 < x < 90$$

Collecting everything on the left

$$\sec 2x - 3\cos 2x - \cos(90 - x) = 0$$

Using the definitions of the trigonometric functions

$$\sec 2x = \frac{1}{\cos 2x}$$

Using the complementary identities

$$\cos(90 - 2x) = \sin 2x$$

$$\frac{1}{\cos 2x} - 3\cos 2x - \sin 2x = 0$$

Dividing by $\cos 2x$

$$\frac{1}{\cos^2 2x} - 3 - \frac{\sin 2x}{\cos 2x} = 0$$

Using the Pythagorean identities

$$\sin^2 2x + \cos^2 2x = 1$$

$$\frac{\sin^2 2x + \cos^2 2x}{\cos^2 2x} - \frac{\sin 2x}{\cos 2x} - 3 = 0$$

Using the definitions of the trigonometric functions

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$

$$\tan^2 2x + 1 - \tan 2x - 3 = 0$$

$$\tan^2 2x - \tan 2x - 2 = 0$$

This is quadratic equation for $\tan 2x$

Solving

$$\tan 2x = \frac{1 \pm \sqrt{1^2 - 4 \times 1 \times (-2)}}{2 \times 1} = \frac{1 \pm 3}{2}, \text{ so}$$

$\tan 2x = 2$ and $\tan 2x = -1$, where $-180^\circ < 2x < 180^\circ$, then

$$2x = 63.43^\circ, 2x = -116.57^\circ, 2x = -45^\circ \text{ so}$$

$$\text{Answer: } x = 31.72^\circ, x = -58.28^\circ, x = -22.5^\circ$$

