Question 1. *Let $A = \{1, 2, 3\}$. What is R if R is the relation $\subset$ on $P(A)$. How can you say that a relation is complete?*

Solution. Recall that $P(A)$ is the set of all subsets of $A = \{1, 2, 3\}$, i. e.

$$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

The relation $\subset$ on $P(A)$ is the relation "to be a proper subset". This means that for all $X, Y \in P(A)$ we have $X \subset Y$ if and only if X is a subset of Y, and there are elements of Y which do not belong to X. For example, $\{1\} \subset \{1, 3\}$, because $\{1\}$ is contained in $\{1, 3\}$ and does not coincide with the whole $\{1, 3\}$. But $\{1\} \not\subset \{2, 3\}$, because $\{1\}$ is not a subset of $\{2, 3\}$. Moreover, $\{1\} \not\subset \{1\}$, because $\{1\}$ is a subset of $\{1\}$, which is not proper.

A relation R on a set A is said to be complete if $R = A \times A$, i. e. for all $a, b \in A$ we have $(a, b) \in R$. It is the maximal possible relation on A in the sense that any relation on A is contained in R. □