Question 1. *Let* $A = \{1, 2, 3\}$*. Describe* R *if* R *is the following relation:*

(a) R *is the relation* $<$ *on* A*;*

(b) R *is the relation* $\geq$ *on* A*;*

(c) R *is the relation* $\subset$ *on* $P(A)$*.*

Solution. (a) R is the relation "to be strictly less than", i. e. $(a, b) \in R$ if and only if a is strictly less that b. We can even write all the pairs (a, b), which belong to R:

$$R = \{(1, 2), (1, 3), (2, 3)\}.$$

However, for example, $(1, 1) \notin R$, because 1 equals 1 and hence it is not strictly less that 1.

(b) R is the relation "to be greater than or equal to", i. e. $(a, b) \in R$ if and only if a is strictly greater than or equal to b. Equivalently, $(a, b) \in R$ if and only if a is not strictly less than b. We see that this relation is the complement of the relation $<$ considered above. So,

$$R = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)\}.$$

(c) Recall that $P(A)$ is the set of all subsets of $A = \{1, 2, 3\}$, i. e.

$$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

The relation $\subset$ on $P(A)$ is the relation "to be a proper subset". This means that for all $X, Y \in P(A)$ we have $X \subset Y$ if and only if X is a subset of Y, and there are elements of Y which do not belong to X. For example, $\{1\} \subset \{1, 3\}$, because $\{1\}$ is contained in $\{1, 3\}$ and does not coincide with the whole $\{1, 3\}$. But $\{1\} \not\subset \{2, 3\}$, because $\{1\}$ is not a subset of $\{2, 3\}$. Moreover, $\{1\} \not\subset \{1\}$, because $\{1\}$ is a subset of $\{1\}$, which is not proper. $\square$