Question 1. *If R is a ring with identity, then prove that $\langle a \rangle_l = \{ta \mid t \in R\} = Ra$.*

Solution. Prove that Ra is a left ideal. Indeed, for any $ta \in Ra$ and for any $r \in R$ we have

$$r \cdot ta = rt \cdot a \in Ra.$$

Furthermore, for arbitrary $t_1 a, t_2 a \in Ra$:

$$t_1 a - t_2 a = (t_1 - t_2)a \in Ra.$$

So, Ra is a subgroup of additive group of R. Obviously, Ra contains a, because $a = 1 \cdot a$. So, $\langle a \rangle_l$, as a minimal left ideal containing a, is a subset of Ra. Conversely, since $a \in \langle a \rangle_l$ and $\langle a \rangle_l$ is a left ideal by definition, then for any $t \in R$ the product ta should belong to $\langle a \rangle_l$. Hence, $Ra \subset \langle a \rangle_l$. Both inclusions mean that $\langle a \rangle_l = \{ta \mid t \in R\} = Ra$. $\square$