

Question 1. Evaluate $\int_C f(z)dz$, $f(z) = \frac{z^2}{z+3}$, $C = \{|z| = 1\}$.

Solution. The function $f(z)$ has the only one singular point in \mathbb{C} : $z_0 = -3$. Since $|z_0| = |-3| = 3 > 1$, we conclude that this point does not belong to the domain $D_\varepsilon = \{|z| < 1 + \varepsilon\}$ for some small $\varepsilon > 0$. Hence, f is holomorphic in D_ε . Since $C \subset D_\varepsilon$, by Cauchy theorem the integral of f along C is zero. Thus, $\int_C f(z)dz = 0$.

Answer: $\int_C f(z)dz = 0$. □