Show that the following equation has a solution of the form $u(x, y) = e^{(ax+by)}$ and find the constants a and b:

$$u_{xx} + u_{yy} = 5e^{(x-2y)}$$

Solution:

$$u_x = \frac{\partial u}{\partial x} = ae^{(ax+by)}$$

$$u_{xx} = \frac{\partial^2 u}{\partial x^2} = a^2 e^{(ax+by)}$$

$$u_y = \frac{\partial u}{\partial y} = be^{(ax+by)}$$

$$u_{yy} = \frac{\partial^2 u}{\partial y^2} = b^2 e^{(ax+by)}$$

$$u_{xx} + u_{yy} = a^2 e^{(ax+by)} + b^2 e^{(ax+by)} = (a^2 + b^2)e^{(ax+by)}$$

$$(a^2 + b^2)e^{(ax+by)} = 5e^{(x-2y)} \rightarrow a = 1, b = -2$$

Answer:$a = 1, b = -2.$