Question #22435 Let U and V be vector spaces over a field F. Let $T\colon U \to V$ is one-one if and only if ...
(A) $\operatorname{rank}(T) = 0$
(B) $\operatorname{rank}(T) = 1$
(C) $\ker(T) = 0$
(D) $\ker(T) = 1$
Please explain

Solution. Let us prove that the wright answer is C. Really, assume that $\ker T = 0$, then if one has $T(u_1) = T(u_2)$, when $u_1 \neq u_2$, then $T(u_1 - u_2) = 0$, since T is linear, so $u_1 - u_2 \in \ker T$, which contradicts the assumption that $\ker T = 0$. Now assume that T is 1-1 mapping. Since T is linear that $T(0) = 0$, and since T is 1-1, then $T(u) = 0$, $u \neq 0$, thus $\ker T = 0$. Hence

Answer C.

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