Question 1. *Use the alternating series test to determine convergence or divergence of the series* $\sum_{i=1}^{\infty}(-1)^{i+1}\frac{i+3}{i^2+10}$.

Solution. The series has the form $\sum_{i=1}^{\infty}(-1)^{i+1}a_i$, where $a_i = \frac{i+3}{i^2+10}$. If we show that the sequence a_i is asymptotically decreasing and tends to 0 as $i \to \infty$, then by Leibnitz rule the series $\sum_{i=1}^{\infty}(-1)^{i+1}a_i$ is convergent. Indeed,

$$\begin{aligned}\frac{i+3}{i^2+10} &= \frac{i+3}{i^2+6i+9-6i-18+19}\\ &= \frac{i+3}{(i+3)^2-6(i+3)+19}\\ &= \frac{1}{i+3+\frac{19}{i+3}-6}\\ &= \frac{1}{b_i-6},\end{aligned}$$

where $b_i = i+3+\frac{19}{i+3}$. It is sufficient to prove that b_i is asymptotically increasing and tends to ∞ as $i \to \infty$. We have

$$\begin{aligned}b_{i+1}-b_i &= \left(i+4+\frac{19}{i+4}\right)-\left(i+3+\frac{19}{i+3}\right)\\ &= 1+19\left(\frac{1}{i+4}-\frac{1}{i+3}\right)\\ &= 1-\frac{19}{(i+3)(i+4)}.\end{aligned}$$

Since $\frac{19}{(i+3)(i+4)} \to 0$ as $i \to \infty$, then starting from some $I \in \mathbb{N}$ we have $1-\frac{19}{(i+3)(i+4)} > 0$ and so $b_{i+1} > b_i$ for $i > I$. This shows that b_i increases, when $i > I$. The fact that it tends to ∞ easily follows from the observation that $i+3 \to \infty$ and $\frac{19}{(i+3)(i+4)} \to 0$ as $i \to \infty$.
Answer: it is convergent by Leibnitz rule. □