Question 1. *If $A = 2i + j + k$, $B = i - 2j + 2k$, and $C = 3i - 4j + 2k$, find the projection of $A + C$ in the direction of B.*

Solution. First of all find $A + C$. We have

$$A+C = (2i+j+k)+(3i-4j+2k) = (2+3)i+(1-4)j+(1+2)k = 5i-3j+3k.$$

The projection can be calculated by the formula

$$pr_B(A + C) = \frac{B(A + C)}{|B|^2}B,$$

where $B(A + C)$ is the scalar product of B and $A + C$, $|B|^2$ is the square of the absolute value of B. Find these values:

$$B(A+C) = (i-2j+2k)(5i-3j+3k) = 1\cdot 5+(-2)\cdot(-3)+2\cdot 3 = 5+6+6 = 17,$$

and

$$|B|^2 = (i - 2j + 2k)^2 = 1^2 + (-2)^2 + 2^2 = 1 + 4 + 4 = 9.$$

Thus,

$$pr_B(A + C) = \frac{17}{9}(i - 2j + 2k) = \frac{17}{9}i - \frac{34}{9}j + \frac{34}{9}k.$$

Answer: $\frac{17}{9}i - \frac{34}{9}j + \frac{34}{9}k$. □